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Rapid Note

Photon-number-state generation with a single two-level atom in a cavity: a proposal

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Abstract. A single two-level atom can be used to prepare an arbitrary photon number state (Fock state) in a high Q cavity . The atom undergoes a controlled succession of interactions with two cavity modes. One of them contains a coherent field. The atom transfers photons one by one from this field to the initially empty second mode. The scheme can be extended to prepare a quantum superposition of the vacuum with a Fock state, a highly non-classical situation. We discuss the feasibility of the experiment with our present Rydberg-atom cavity QED set-up.

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Quantum field states engineering has become a very active domain in quantum optics. Squeezed states or single photon Fock states are now available in various experimental conditions. However, Fock states with more than one photon have not yet been prepared. They could be used for various applications, such as quantum cryptography or communication [1]. The high level of control of the matter-field interaction achieved in cavity QED [2] makes it a very interesting background for such experiments and various Fock state generation schemes have already been proposed in this context.

A first approach to the generation of Fock states relies on a quantum non demolition measurement of the photon number, projecting the cavity state onto a photon number state [3]. The photon number value obtained in a given experiment is however unpredictable.

In proper conditions, a microlaser [4] or micromaser [5] generates directly a Fock state. The micromaser, for instance, operating in a trapping state, provides a highly sub-Poissonian field but requires an excellent control of dissipation, thermal noise and atomic beam fluctuations. Other proposals use a well determined number of atoms emitting one photon each in the cavity [6,7]. These methods require a very high detection efficiency, not achieved experimentally yet, to determine precisely the number of emitting atoms. A method relying on the adiabatic emission of photons by an ensemble of atoms crossing simultaneously the cavity [8] suffers from the same kind of limitation.

Proposals using a single atom escape the detection efficiency problem at the price, however, of complications in the required atomic level scheme or field mode structure. An adiabatic transfer, mapping atomic Zeeman states populations onto a cavity field [9] can in principle prepare any superposition of Fock states. Photons can be "pumped" one by one from classical fields into the quantized cavity mode by a single three-level atom [10]. This method, which can be extended to generate an arbitrary superposition of Fock states, relies on a Raman transition scheme involving four field modes.

In this letter, we propose a very simple way of generating a given photon-number state. It relies on a "minimal" configuration involving a single two-level atom and only two field modes, a "photon reservoir" prepared in a coherent state and the quantum mode storing the Fock state. This method is also of great practical interest, since it is compatible with the properties of circular Rydberg states and high Q superconducting cavities used in microwave cavity-QED experiments [11–14]. The complex atom and field energy level structures required by the previously proposed methods could not be implemented in this context. Moreover, our method can be straightforwardly generalized to prepare highly non-classical superpositions of an arbitrary Fock state with the vacuum.

The atom, whose resonant frequency is controlled by Stark effect, crosses the cavity while interacting

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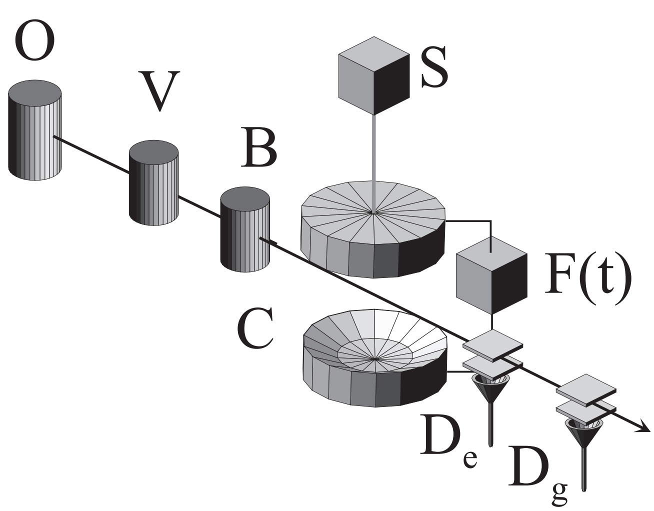

Fig. 1. Sketch of the proposed experimental set-up.

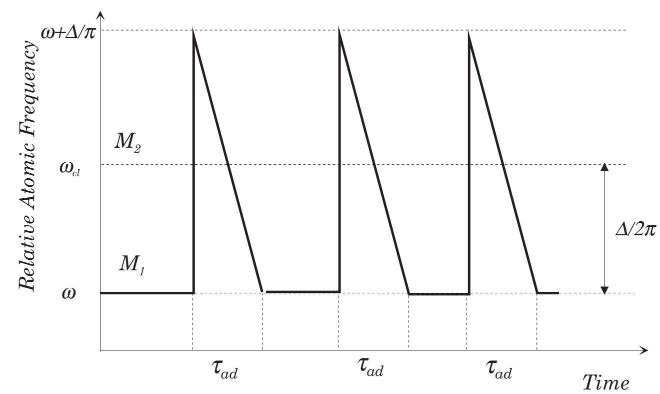

Fig. 2. Scheme of the proposed timing. The atomic transition frequency, controlled through the Stark field $F(t)$, is plotted pictorially versus time. It is tuned alternatively to modes M_1 and M_2, separated by $\Delta/2\pi$.

alternatively with the two modes. An adiabatic process is used to excite efficiently the atom in the "classical" coherent field. This excitation is then deposited in the "quantum" field.

The proposed experimental set-up is sketched in Figure 1. It is the one already used in previous experiments [11–14]. A single circular Rydberg atom is prepared in box B into level e (principal quantum number 51) at a given time by excitation of the atomic beam effusing from oven O. The atomic velocity v is selected prior to the Rydberg state excitation in zone V. It can be adjusted between 100 and 400 m/s. The atomic position is therefore known at any time during the atomic transit through the set-up. The atom interacts with the superconducting cavity C, made of two spherical niobium mirrors in a Fabry-Perot configuration.

The cavity sustains two orthogonally polarized TEM_{900} modes, M_1 and M_2. The slight ellipticity of the mirrors lifts their degeneracy. The frequencies are ω for M_1 and $\omega_{\mathrm{cl}} > \omega$ for M_2. The frequency difference, $\Delta/2\pi = (\omega_{\mathrm{cl}} - \omega)/2\pi$, ranges from 100 kHz to 2 MHz for different mirror pairs presently available. Both modes share the same Gaussian geometry, with a $w = 6$ mm waist. A classical source S prepares initially in M_2 a coherent field $|\alpha\rangle$ with an average photon number $\overline{n} = |\alpha|^2$, while M_1 is left empty (the cavity temperature, 0.6 K, makes thermal radiation negligible). Both modes are close to resonance with the transition between e and the lower circular state g (principal quantum number 50, $e \to g$ transition frequency 51.099 GHz). A time-varying electric field $F(t)$ applied between the two mirrors is used to tune alternatively the atomic frequency ω_{at} by Stark effect on resonance with each mode. After its interaction with the cavity, the atom is finally detected downstream in one of the field-ionization detectors D_{e} (for level e) or D_{g} (for level g).

The time sequence is depicted in Figure 2. The atom, injected into the cavity in its excited state $|e\rangle$, is tuned to resonance with M_1 ($\omega_{\mathrm{at}} = \omega$) during the time required to undergo a π pulse. The single photon Rabi frequency Ω at cavity center is 50 kHz. The actual interaction time with M_1 differs from π/Ω however because the atom explores the Gaussian profile of the field. This time is perfectly determined from the knowledge of the atomic trajectory. When one photon exactly is deposited in the cavity (combined atom-cavity-mode-M_1 state $|g,1\rangle$), the atomic resonance frequency is detuned suddenly by 2Δ to reach $\omega_{\mathrm{at}} = \omega_{\mathrm{cl}} + \Delta$. The action of the classical field in M_2 on the atomic state during this diabatic switching is negligible. By slowly decreasing then the atomic frequency ω_{at} and passing through the resonance $\omega_{\mathrm{at}} = \omega_{\mathrm{cl}}$, the atom is adiabatically transferred into state e. The combined atom-mode M_1 state is then $|e,1\rangle$.

The resonant interaction ($\omega_{\mathrm{at}} = \omega$) resumes for a shorter interaction time (taking into account the fact that the Rabi frequency in a single photon field is $\sqrt{2}$ larger than in the vacuum [11]). The whole sequence can be repeated as many times as required to pump more photons in M_1. At each resonant step with M_1, the variation of the Rabi frequency with the photon number is taken into account. Finally, the atom exits the cavity, leaving M_1 in a Fock state. In principle, we may reach an arbitrary photon number. There are however practical limitations on the accessible interaction times. First, one cannot select an arbitrarily small atomic velocity. Second, the loss of a photon due to cavity relaxation tends to blur the final state at long times. The total interaction time must be kept much shorter than the photon lifetime in the cavity.

Let us now discuss the parameter optimization taking into account the experimental constraints. The adiabatic transfer is efficient provided $\Omega_{\mathrm{cl}}^2 \tau_{\mathrm{ad}}/\Delta > 1$ where τ_{ad} is the adiabatic passage time (see Fig. 2) and $\Omega_{\mathrm{cl}} = \Omega\sqrt{\overline{n}+1}$ the atom-classical field average coupling at resonance. The frequency mismatch Δ is determined by the cavity geometry. To keep τ_{ad} as short as possible, we need thus a large coupling Ω_{cl} and hence a large average photon number in M_2. This strong field in M_2 may influence the resonant interaction between the atom and M_1. To keep this effect small, we must fulfill the

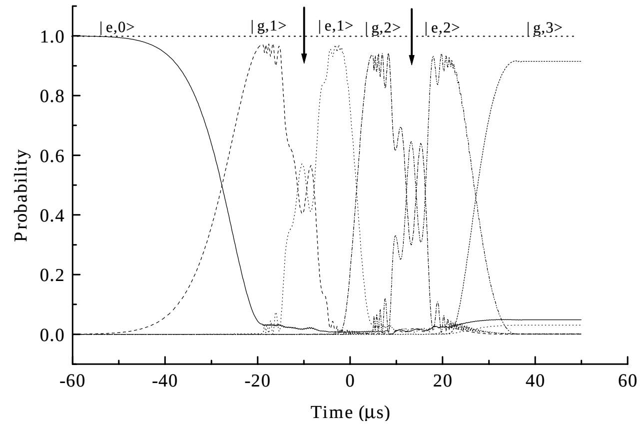

Fig. 3. Atom-cavity states weights versus time for a preparation sequence yielding a three photon Fock state. The central times of the adiabatic passage sequences are indicated by arrows. Note the oscillations of the states populations around these times, when the atom is at resonance with mode M_2. The target state $|g,3\rangle$ is reached with a 0.92 probability. Atomic velocity is 200 m/s.

condition $\Omega_{cl}/\Delta \ll 1$. A satisfactory compromise can be found for the microwave cavity setup used in our laboratory. Let us assume a frequency mismatch $\Delta = 4\pi$ MHz. We can choose Ω_{cl} in the 1 rad/μs range, corresponding to $\overline{n}$ =30-40. With these parameters, the adiabatic passage is efficient for $\tau_{ad} = 10$ μs. For an atomic velocity $v = 200$ m/s, the total interaction time in the cavity is about 40-50 μs. During this time we can realistically feed up to 3 photons into the mode.

The efficiency of the adiabatic rapid passage in the limited interaction time τ_{ad} can be further increased by tailoring the time variation of the atomic frequency. Instead of a linear variation, depicted in Figure 2 for sake of clarity, we have determined, through numerical simulations, that a cubic variation of the detuning versus time ($\omega_{at} - \omega_{cl} = \Delta(1 - 2(t - t_0/\tau_{ad})^3)$, t_0 corresponding to the beginning of the adiabatic passage sequence) provides much better results. The probability of atomic excitation, only 90% for the linear variation, increases up to close to 100% in this case, which could be easily implemented experimentally through a proper control of the Stark electric field $F(t)$.

We have checked the scheme by complete numerical simulations. The evolution of the amplitudes of the different states are shown in Figure 3 (ideal lossless cavity, atomic velocity 200 m/s). The states $|g,1\rangle$, $|e,1\rangle$, $|g,2\rangle$, $|e,2\rangle$ and $|g,3\rangle$, are reached successively with weights 97.1%, 94%, 93.6%, 92% and finally 91.5%. This corresponds to an almost perfect preparation of a Fock state. The purity of the state can be further increased by using the information provided by the atomic detection. With a 91.8% probability, the atom is counted in D_g. In this case, the field is projected onto a state containing the $|3\rangle$ Fock state with 99.7% weight. The scheme produces an almost perfect Fock state in a single conditional measurement.

The total flight time of the atom in the apparatus is in the 300-400 μs range. The atomic radiative lifetime (30 ms) plays therefore no role in the experiment. The photon lifetime in the cavity T_r, up to about 3 ms (cavity quality factor in the 10^9 range), might introduce non-negligible dissipation effects. To estimate them, we performed a Monte-Carlo wavefunction simulation [15].

Random quantum jumps corresponding to photon loss from the cavity are introduced into the coherent evolution. The calculated final probabilities of the atom-cavity states are summarized in Table 1 (first line: no relaxation with the conditions described above, second line: cavity mode damping time 3 ms). Relaxation reduces the target state $|g,3\rangle$ weight down to 85.7%. Accordingly, the probability of the $|g,2\rangle$ state is increased, from 0.1% in the absence of relaxation, up to 4.8%. The method is therefore quite robust against dissipation. Note that the sum of the $|g,2\rangle$ and $|g,3\rangle$ states weights corresponds approximately to the 91.5% weight of state $|g,3\rangle$ in a relaxation-free evolution, reflecting the fact that the probability of two consecutive photon losses during the evolution is negligible.

The Monte-Carlo method is also well suited to account for imperfections in the atomic position determination. In fact, the atomic position is known inside C with a 0.5 mm accuracy at best, due to the size of the excitation lasers and to the residual velocity dispersion. The coupling to the cavity at a given time for different realizations of the experiment, slightly blurring the final field state. The final weights taking into account this effect are given in the third line of Table 1. The weight of the target state $|g,3\rangle$ is reduced further down to 76.5%.

Once again, the information gained by detecting the internal state of the atom in D_e or D_g, can be used to improve the quality of the final state. Table 2 gives, in the first column, the probability to detect the atom in state g. The second column gives the 3 photon Fock state weight in the cavity, conditioned to the detection of the atom in g. Even all noises included, the probability of the three-photon Fock state is as high as 88.8%.

A simple modification of the set-up can lead to the production of an highly non-classical field state, quantum superposition of the vacuum with an n photons Fock state. Let us consider the circular Rydberg level i with principal quantum number 52. It is coupled to e by a dipole transition, whose frequency is very different from the one of the $e \rightarrow g$ transition, nearly resonant with the cavity. Before entering in C, the atom is prepared in a superposition $(|e\rangle + |i\rangle)/\sqrt{2}$ through the interaction with a resonant classical microwave field contained in a low-Q cavity R_1 (not shown in Fig. 1 for sake of simplicity). The Fock state preparation time sequence is then applied to the atom in the high Q cavity. The linearity of the evolution leads to a superposition of two final states. The atom has either remained in the non-resonant level i, leaving the cavity empty, or evolved from e to g, feeding an n photon Fock state in M_1. A last adiabatic passage in mode M_2 can then be used before the atom exits the cavity to promote again the atom in level e. The atom+cavity state at the end of the interaction is thus: $(|e,n\rangle + |i,0\rangle)/\sqrt{2}$.

Table 1. The probabilities (in percentage) of the atom-cavity states at the end of the evolution. The three lines correspond to evolution without dissipation, with dissipation only (cavity damping time $T_r = 3$ ms), and with dissipation and position dispersion (0.5 mm).

	$\lvert g,0\rangle$	$\lvert e,0\rangle$	$\lvert g,1\rangle$	$\lvert e,1\rangle$	$\lvert g,2\rangle$	$\lvert e,2\rangle$	$\lvert g,3\rangle$	$\lvert e,3\rangle$
without dissipation	0.2	4.9	0.0	3.1	0.1	0.0	91.5	0.2
with dissipation only	0.6	5.1	0.2	0.4	4.8	0.0	85.7	0.2
with position dispersion	1.2	6.7	3.6	3.6	4.8	3.4	76.5	0.2

Table 2. The probabilities (in percentage) of detecting the atom in state $|g\rangle$ and the weight of the $|3\rangle$ Fock state in the cavity, conditioned to the atomic detection in g. The three cases of table I are considered.

	$\lvert g\rangle$	$\lvert g,3\rangle$
without dissipation	91.8	99.7
with dissipation	91.3	93.9
with all noises	86.1	88.8

A direct detection of the atomic state at the exit of the cavity would project the field either on an n photons state or on vacuum. A field quantum superposition can be preserved only if the information about the atomic state in C is “erased”. This can be performed by mixing again levels i and e after the interaction with C in another low Q cavity R_2 identical to R_1. The atom undergoes in R_2 the unitary transformation $|i\rangle \to (|i\rangle + |e\rangle)/\sqrt{2}$ and $|e\rangle \to (-|i\rangle + |e\rangle)/\sqrt{2}$. The atom-cavity state after R_2 is thus:

$$\frac{1}{2}\left[|i\rangle(|0\rangle - |n\rangle) + |e\rangle(|0\rangle + |n\rangle)\right]. \tag{1}$$

The detection of the atom in level e or i projects the field on the state $(|0\rangle + |n\rangle)/\sqrt{2}$ or $(|0\rangle - |n\rangle)/\sqrt{2}$ respectively.

This state is analogous to the “amplitude Schrödinger cat”, superposition of the vacuum and of a coherent state, produced by a quantum switch arrangement [16]. Since it implies a Fock state, the decoherence of such a mesoscopic quantum superposition can be trivially calculated and interpreted. The decoherence time is exactly T_r/n, even for low n values. The escape of the first photon in the environment is enough to determine with certainty whether there is a non vanishing field in the cavity and to spoil the quantum superposition [17]. This law is much simpler than the one describing the decoherence of a superposition of coherent states, especially for low average photon numbers. Observing the decoherence of such a state would be an interesting goal. The coherence of the superposition in C could be tested by a direct mapping of the field's Wigner function [18].

We have proposed a realistic scheme to prepare with a high accuracy a Fock state in a cavity. The scheme could reach photon numbers of about 5 by using slow atoms in the beam. The corresponding experiment is in progress in our laboratory. Extension to the generation of highly non-classical “Schrödinger cat” states is also possible.

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